International Journal of Materials, Methods and Technologies
Vol. 1, No. 7, August 2013, PP: 103 -115, ISSN: 2327- 0322 (Online)
Available online at http://ijmmt.com/

Research article

MATHEMATICAL MODELING AND SIMULATION TO PREDICT THE TRANSPORT OF DIPLOCOCCIC IN HOMOGENEOUS UNCONFINED AQUIFER IN PORT HARCOURT METROPOLIS NIGER DELTA OF NIGERIA

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Abstract

mathematical modeling and simulation to predict the transport of diplococcic in homogeneous unconfined aquifer has been carried out, this study were carried out to monitor the rate of concentration at various formation to ground water aquifers, the derived model generated theoretical values, the model values were compared with experimental values, both parameters developed a faviourable fit, the expressed values shows that the concentration of the microbes experienced lag phase in some formation and high concentration were observed in some formation as presented in the figures, this condition can be attributed to change in concentration with respect to distance, formation variation were found to influence the system base on the structural deposition of the soil, inhibitors influence were found also to have played a major role on the stationary condition of the microbes in some formation, the study is imperative because it has developed a conceptual frame work for practicing professionals to have a stipulated standard to applied for ground design and construction in the study area.

Keywords: modeling and simulation of diplococcic transport and homogeneous fine sand

1. Introduction

The intention of coastal aquifer management is the same as for other aquifer systems – to achieve a sustainable use of groundwater, coordinated with the use of other water resources, to meet part of the demand for water by supplying water of adequate quality, in the place at the right time, respecting environmental and habitat restrictions [Custodio, 2005]. The main additional items to be considered are the risk of salinization and water quality degradation in relation with possible accumulation of manmade contaminants in areas of low hydraulic gradient and flow pattern forming a closed area due to groundwater abstraction conditions [Arbam 1999]. Often these risks do not result in immediate threats, but the results may be delayed for a long time. This means that coastal aquifer management should rely on conservation and protection measures [Schiedeger and lindinger, 2004, Omotsola and Adegoke, 1981]. Coastal groundwater resources are increasingly a critical component of available freshwater in Nigeria, in a national setting of rising population density in coastal margins. The enhanced dependency on coastal groundwater has resulted in symptoms of over-extraction, namely seawater intrusion (i.e., the landward encroachment of saline groundwater). The seawater intrusion threat to freshwater supplies has already led to groundwater management response in some regions of Nigeria, with extensive investigation and the construction of monitoring boreholes. Coastal aquifer management in Nigeria needs to account not only for existing threats to freshwater resources and groundwater-dependent ecosystems but also for the influences of climate change which is expected to produce modified groundwater recharge and rising ocean levels. Effective coastal management must be based on a solid scientific foundation, taking into account the limitations of natural systems, while balancing and integrating the demands of the various sectors which depend on these systems for their livelihood The coastline of Nigeria is about 1000km long on the Gulf of Guinea, bordering eight states of Lagos, Ogun, Ondo, Delta, Bayelsa, Rivers, Akwa Ibom and Cross River. While the first four states are west of the River Niger, the last three states are east of the Niger with the last, Bayelsa State, straddling the river. Geologically, Coastal Nigeria is covered by two sedimentary basins, the Keta basin and Niger Delta basin. The Keta basin (also called the Benin- or Dahomey basin in Nigerian literature) is a transboundary basin that extends from Ghana through Togo and Benin to Nigeria. The Niger Delta basin is separated by the Okitipupa Ridge [Hudake,1999]. The Keta Basin constitutes part of a system of West African margin developed during a brief period of rifting in the late Jurassic to Early Cretaceous, associated with the Benin Trough Complex. It was accompanied by an extended period of thermally induced basin subsidence through the Middle to Upper Cretaceous to Tertiary times as the South American and African plate entered a drift phase. The onshore portion of the basin covers a broad arc-shaped profile approximately 600km2, attaining a maximum width of 65km at the basin axis along the Nigerian border with the Republic of Benin. It narrows to about 25km west and eastwards. It is along its north eastern fringe (the Okitipupa Structure) that a band of tar sand (oil sand) and bitumen seepage occurs [Moline,et al 1998, Hudak,1999]. A quantity of text materials exist on diverse processes of proper sitting of monitoring wells based on the character of groundwater flow, and dimensions of the polluted field. [Moline et al,1988] Devised a graphical heuristic for locating up gradient groundwater monitoring wells near landfills, and observed that it can be adapted to no homogeneous flow fields, heterogeneous transport limitations, and irregularly shaped landfills oriented at various angles to the direction of groundwater flow. [Hudak,2001] observed that the need for satisfactory characterization of spatial and temporal variations of groundwater flow are for suitable position and construction of monitoring wells, timing of ground water monitoring, and evaluation of exposure risk and contaminant flux in support of remedial decision-making. [Macdonald and hurbough, 1988]. observed that heterogeneous network, groundwater monitoring wells clustered near the down-gradient corner of a landfill registered 100% detection efficiency. This strategy was effective because the cut-off wall induced convergent groundwater flow beneath the landfill. This study suggests that distorted hydraulic head fields induced by partial cut-off walls will be considered when designing detection monitoring networks at landfills. Advances in numerical modeling software such as MODFLOW [Nwachukwu and Alinor, 2002] has simplified the study of groundwater flow, which in this paper is been extended to selecting sites for groundwater quality monitoring wells. Spilling the occupational wastes of mechanics on the ground has been a common practice in the region for over 25 years [Ibe and Njoku,1999]. There is no enforced regulation for the disposal of municipal solid waste and sewage. Solid waste is disposed of by open dumping at several locations [Ibe and Njoku, 1999, ibe,et al 2007,]. Sewage evacuated from soak-away pits present in virtually every household, is mixed with soil, and used as manure on farm lands around the southwest flank in the area.

2. Materials and Method

Column experiments were also performed using soil samples for several borehole locations, the soil samples were collected at intervals of three metres each (3m). An diplococcic solute was introduced at the top of the column and effluents from the lower end of the column were collected and analyzed for diplococcic and the effluent at the down of the column were collected at different days, analysis, velocity of the transport were monitored at different days. Finally, the results were collected to be compared with the theoretical values.

3. Theoretical background

$$\frac{\phi\partial C}{\partial t} = \frac{\partial C}{\partial x}\left[\phi D\frac{\partial C}{\partial x}\right] - V\frac{\partial C}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Applying Laplace transformation into equation (1) we have

$$\frac{\partial C}{\partial t} = SC_{(t)} - C_{(o)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\partial C}{\partial x} = SC_{(x)} - C_{(o)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\frac{\partial C}{\partial x} = SC_{(x)} - C_{(o)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$\frac{\partial C}{\partial x} = SC_{(x)} - C_{(o)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$C \;=\; C_{(o)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \qquad (6)$$

Substituting equation (2), (3), (4), (5) and (6) into equation (7) yields

$$\phi\left[SC_{(t)} \;=\; SC_{(x)} \;-\; C_{(o)}\right] - \phi D\left[SC_{(x)} \;=\; SC_{(x)} \;-\; C_{(x)}\right] - V\,C_{(o)} \qquad \ldots\ldots \qquad (7)$$

$$\phi\,SC_{(x)} \;-\; \phi S^{1}{}_{(x)} \;-\; C_{(o)} - \phi D\,SC_{(o)} \;-\phi D\,C_{(o)} \;-V\,C_{(o)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots$$

(8)

Considering the following boundary as:

At $t = 0,\; C^{1}{}_{(o)} = C_{o} \;=\; 0$ ……………………………. (9)

We have

$$C_{(x)}\left(\phi S \;-\; \phi S \;-\; \phi DS\right) \;=\; 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \qquad (10)$$

$$C_{(x)} \;\neq\; 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \qquad (11)$$

Considering the boundary condition

at $t > 0,\; C(^{1}{}_{(o)} \;=\; C_{(o)} \;=\; C_{o}$ ………………………….…. (12)

$$SC_{(t)} \;-\; \phi DS_{(x)} \;-\; VC_{(x)} \;=\; \phi\,SC_{o} \;+\; \phi D\,C_{o} \;+\; V\,C_{o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (13)$$

$$\left[\phi S - \;\phi D \;-\; V\right]C_{(x)} \;=\; \left[\phi\,S \;+\; \phi \;+\; \phi D\,\right]C_{o} \qquad \ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots \qquad (14)$$

$$C_{(x)} \;=\; \frac{\phi\,S \;+\; \phi \;+\; \phi D}{\phi S \;-\; \phi DS \;+\; V}\,C_{o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (15)$$

Applying quadratic expression, we have

$$S \;=\; \frac{-\,b \,\pm\, \sqrt{b^{2} - 4ac}}{2a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (16)$$

Where $a = \phi$, $b = \phi D$, $c = V$

$$\frac{-\,\phi D\,\sqrt{-\,\phi D^{2} + 4\phi v}}{2\phi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (17)$$

$$C_{(x)} = A \exp \frac{\lfloor -\phi D + \sqrt{-\phi D^2 + 4\phi v} \rfloor}{2\phi} t = - \exp \frac{\lfloor -\phi D + \sqrt{-\phi D^2 + 4\phi v} \rfloor}{2\phi} t \quad \ldots\ldots \qquad (18)$$

Subjecting equation (17) to the following boundary condition and initial value condition

$$x = 0,\ C_{(o)} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (19)$$

We have $B = -1 \; and \; A = 1$ ……………………………… (20)

So that our particular solution, will be in this form

$$C_{(x)} = \exp\left[-\phi D + \left(\phi D^2 - 4\phi v\right)^{1/2} \right] x - \exp \frac{\lfloor -\phi D + \sqrt{-\phi D^2 + 4\phi v} \rfloor}{2\phi} t \quad \ldots\ldots \qquad (21)$$

But $e^{x} - e^{-x} = 2 Sin\, x$

Therefore, the expression of (20) can be of this form

$$C_{(x)} = 2 Sin \left[\phi D^2 + \left(\phi D^2 + 4\phi v\right)^{1/2} \right] x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (22)$$

But if $x = \frac{V}{t}$

Therefore, the model can be expressed as:

$$C_{(x)} = 2 Sin \left[\phi D^2 + \left(\phi D^2 + 4\phi v\right)^{1/2} \right] \frac{V}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (23)$$

Again $\frac{V}{t} = x$

We have

$$C_{(x)} = 2 Sin \left[\phi D + \left(\phi D^2 + 4\phi v\right)^{1/2} \right] x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (24)$$

Considering (22) and (23) yield

$$\boxed{C_{(x,t)} = 2 Sin \left[\phi D + \left(\phi D^2 + 4\phi v\right)^{1/2} \right] x + 2 Sin \left[\phi D + \left(\phi D^2 + 4\phi v\right)^{1/2} \right] x} \quad \ldots. \qquad (25)$$

3 Results and Discussion

Mathematical modeling and simulation to predict the transport of diplococcic in homogeneous unconfined aquifer are presented in table and figures bellow.

Table 1: Comparison of Theoretical and experimental Values of Diplococcic at various depths

Depth	Theoretical values	Experimental values
3	2.78	2.66
6	3.98	3.87
9	2.96	2.88
12	2.50E-01	2.45E-01
15	-4.91E-03	-4.47E-03
18	-3.98	-3.78
21	-3.13	-3.17
24	-0.51	-0.49
27	2.38	2.24
30	3.94	3.86
33	3.28	3.21
36	0.77	0.67

Table 2 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

Depth	Theoretical values	Experimental values
3	3.59	3.45
6	3.16	3.24
9	-0.81	-0.78
12	-3.37	-3.34
15	-2.6	-2.63
18	1.57	1.45
21	3.99	3.45
24	1.94	1.89
27	-2.28	-2.22
30	-3.95	-3.89
33	-1.19	-1.21
36	-2.89	-2.78

Table 3 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

Depth	Theoretical values	Experimental values
3	0.9	0.88

6	1.77	1.66
9	2.54	2.51
12	3.17	3.24
15	3.64	3.57
18	3.92	3.88
21	3.99	4.11
24	3.86	3.78
27	3.52	3.55
30	3	3.11
33	2.32	2.24
36	-1.51	-1.48

Table 4 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

Depth	Theoretical values	Experimental values
3	2.14	2.26
6	3.61	3.55
9	3.96	3.89
12	3.08	3.11
15	1.23	1.12
18	-4.49	-4.15
21	-2.29	-2.34
24	-3.92	-3.88
27	2.35	2.24
30	0.25	0.27
33	1.92	1.89
36	2.11	2.09

Table 3 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

Depth	Theoretical values	Experimental values
3	0.2	0.23
6	0.21	0.25
9	0.31	0.34
12	0.41	0.45
15	0.52	0.55
18	0.62	0.66
21	0.72	0.69
24	0.83	0.78
27	0.93	0.89

30	**1.03**	**1.06**
33	**1.13**	**1.15**
36	**1.23**	**1.28**

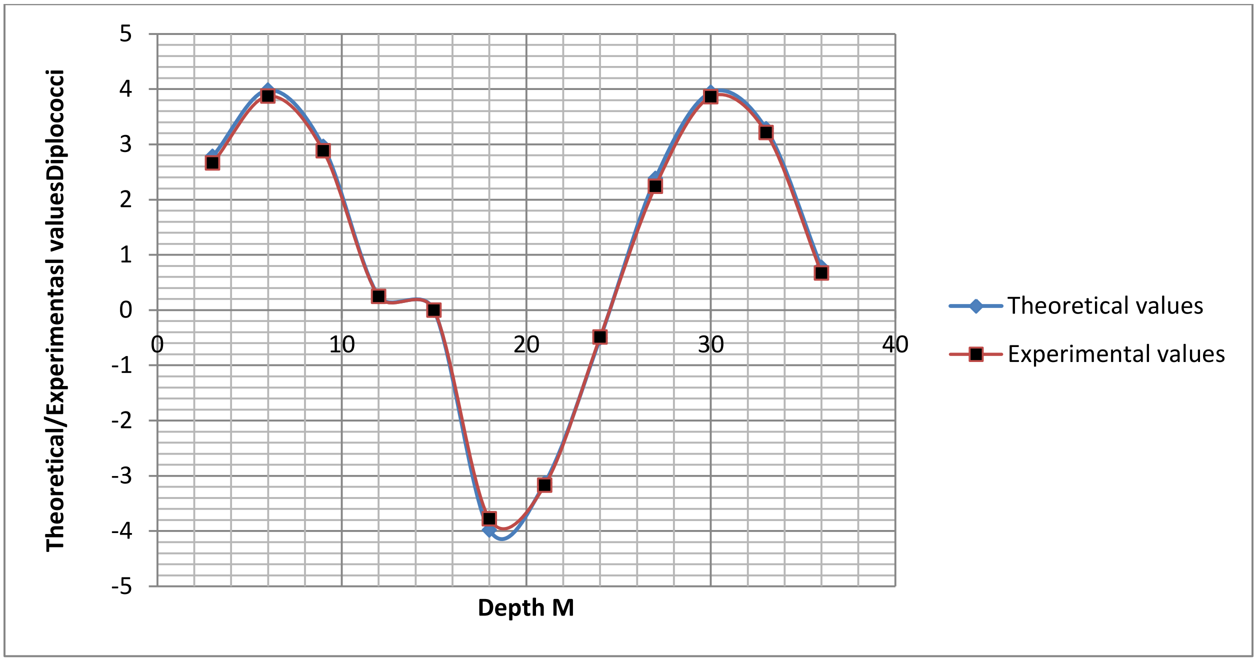

Figure 1 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

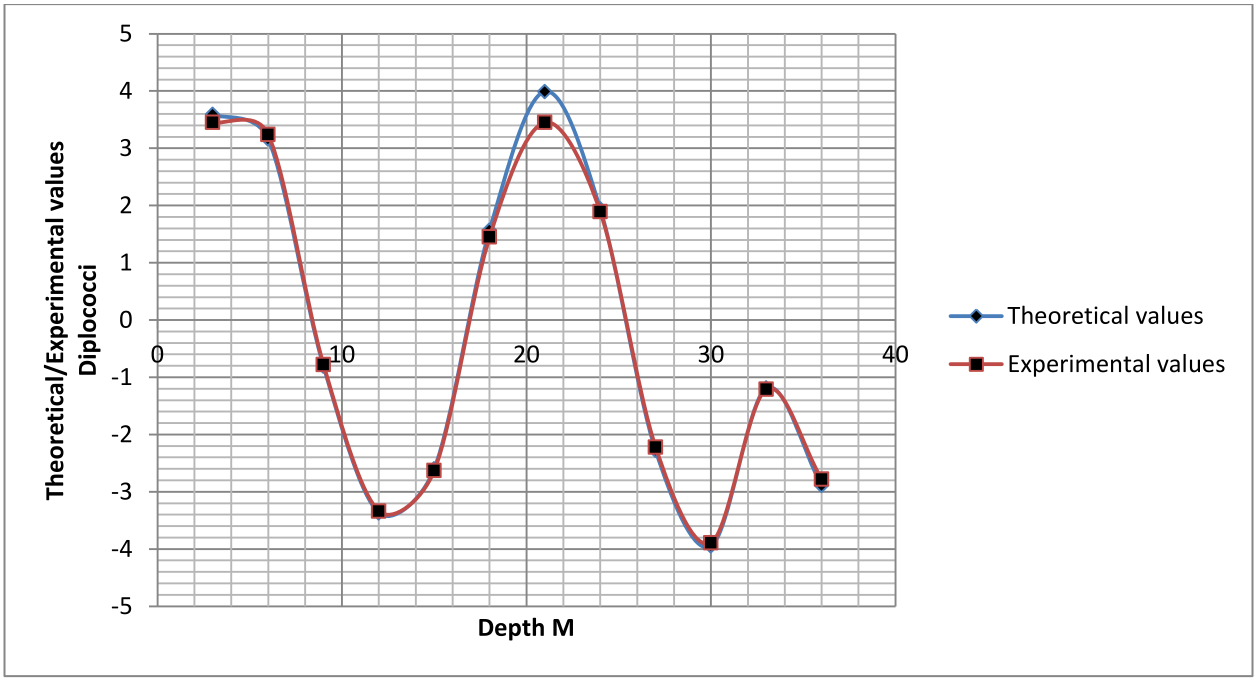

Figure 2 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

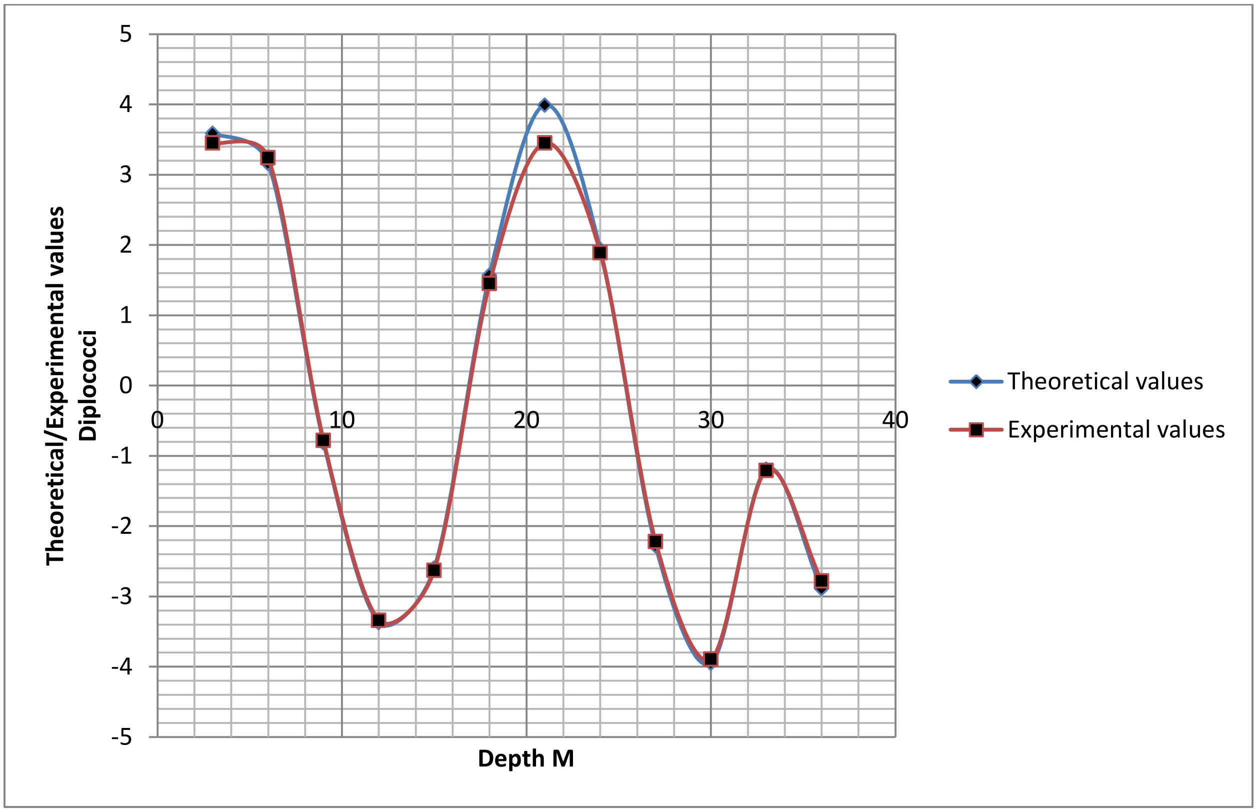

Figure 3 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

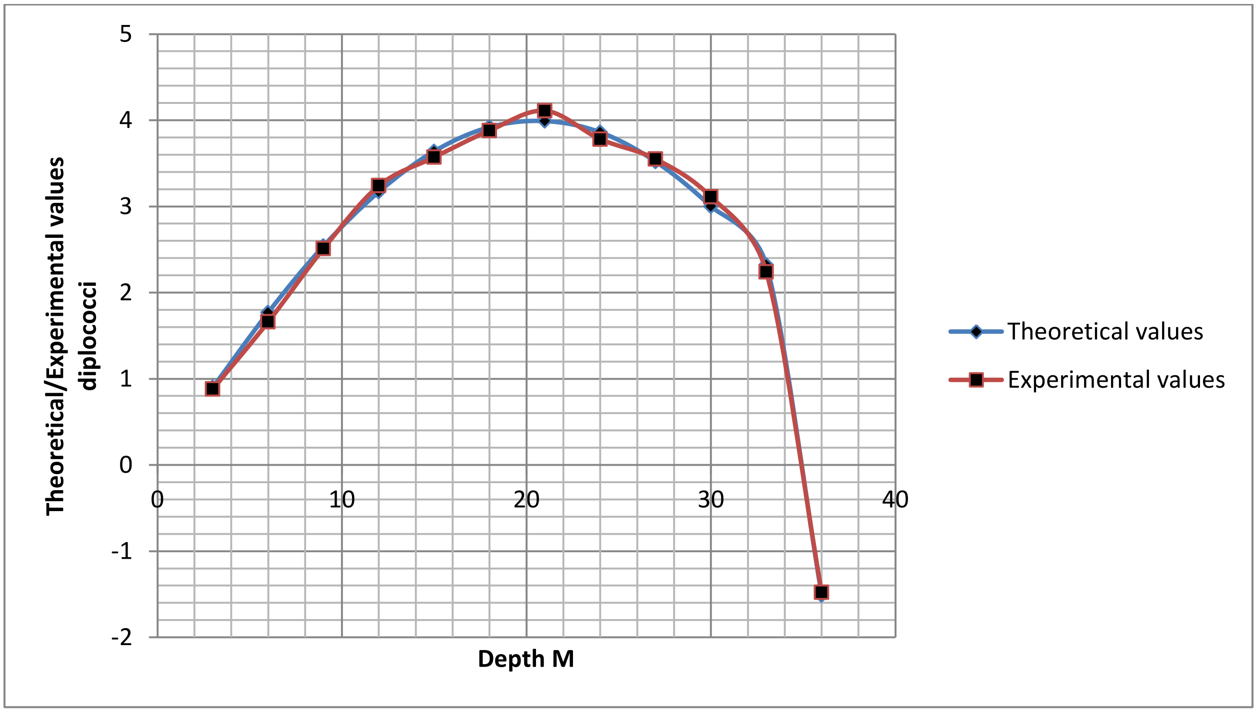

Figure 4 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

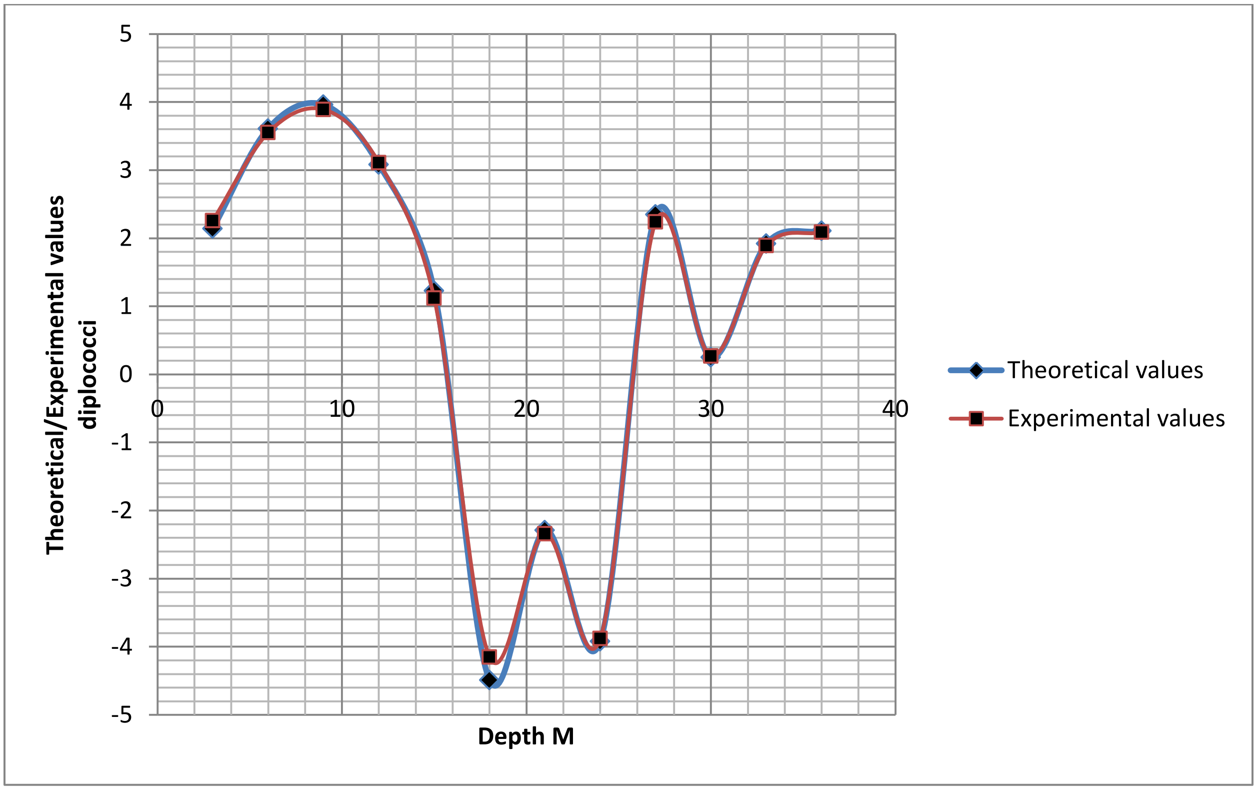

Figure 5 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

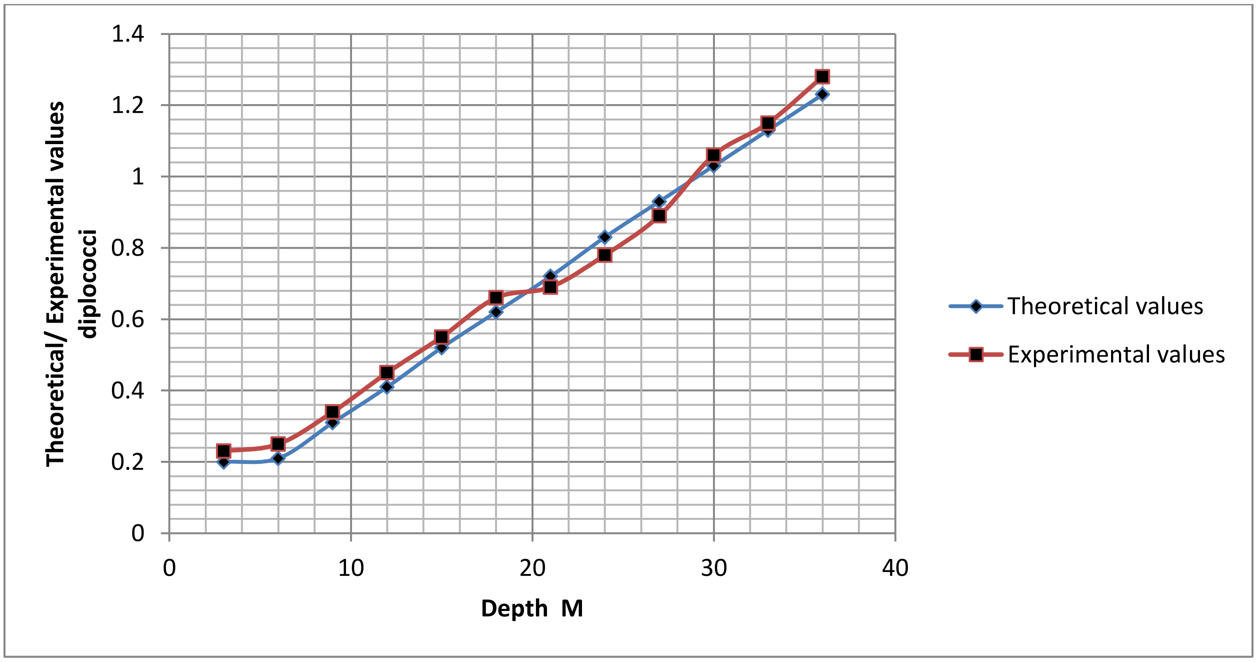

Figure 6 : Comparison of Theoretical and experimental Values of Diplococcic at various depths

Figure 1 shows that the microbes rapidly increased at ten and twenty days, sudden decrease where observed between six and \nine metres, lag phase condition where experienced between twelve and twenty on metres , sudden increase where observed from twenty four to thirty six metres , while experimental values observed similar condition the highest degree of concentration where recorded at three metres, lag phase where observed between twelve and twenty one metres , while the optimum level where recorded at thirty metres, finally slight decrease where observed at thirty six metres Figure 2 experienced rapid increase at three metres sudden decrease where observed between six and to another deposition where lag phase where also observed twelve metres where lag phase where recorded fluctuating down to thirty-six metres, the optimum level where recorded at twenty on metres .while the experimental values experienced speedy increase at three metres and unexpected decrease were observed from six to twenty-one metres where the optimum level where recorded, similarly a sudden decrease where also recorded between twenty-four and thirty metres, and finally it experienced slight lag at thirty six metres. Figure 3 experienced similar condition like figure 2, speedy increase where observed at three metres , while sudden decrease where observed to point where the microbes experienced lag phase, and it fluctuate to where the optimum values where recorded at twenty one metres and it fluctuate down to where it experience another lag phase between from thirty to thirty-six metres. While experimental value maintained the same pace of concentration, where lag phase was experienced between nine and fifteen metres. Sudden fluctuation were experienced recording an optimum value at twenty-one metres, on the process, speedy degradations were observed between twenty-four and thirty metres where another lag phase was recorded. It finally experienced slight increase from thirty to thirty-six metres. Figure 4 observed a gradual increase from three metres to where the optimum value was recorded at twenty-one metres. Gradual degradation was also observed from twenty-four to thirty-six metres where lag phase was observed. Experimental value observed similar condition; gradual increase was experienced recording an optimum value at twenty-one metres and finally decreasing with distance to where stationary phase was experienced at thirty-six metres. Figure 5 observed a speedy increase at three metres, it expressed a gradual increase between six and nine metres while sudden decrease were recorded to where stationary phase were observed between eighteen and twenty-four metres. Rapid increase was also experienced between twenty-four and twenty-seven metres where a slight lag phase was observed. Finally, it expressed slight increase between thirty-three and thirty-six metres while experimental values experienced speedy increase at three metres observing a gradual increase between six and nine metres. Sudden degradation were expressed where lag phase was observed between eighteen and four metres and it fluctuates down to thirty-six metres with a slight stationary phase between twenty-seven to thirty-six metres. Figure 6 expressed a gradual increase where the lowest concentration was recorded at three metres and optimum values were observed at thirty-six metres. The experimental value developed a constant concentration between three and six metres, gradual increase was observed with a slight fluctuation to where the optimum values were recorded at thirty-six metres. In general concept, the study experienced a lot of lag phases, these can be attributed to high deposition of inhibitors from heavy metals including change of concentration with respect to distance.

5. Conclusion

Diplococcic is one of the microbes found in water environment, this microbes are a serious threat to water that are sources of human utilizations at the study location, this type of water pollution is a threat to human life, and it call for a thorough study, mathematical model were develop to monitor the rate of transport of the type microbial specie in the study area, possibly finding out there rate of migration and concentration at different formation and depths. From the study carried out, the model developed expressed the rate of concentration, were by the microbes were found to develop lag phase in some formation, and expressed rapid increase in some formation, the condition of the microbial transport become heterogeneous in a high percentage, this condition implies that quality ground water in the study area definitely need thorough design considering the nature of soil stratification as it influenced the deposition of the microbes at different formation. Furthermore the quality of ground water in the study location need a design criteria to be integrated in the design and construction of water well to avoid abortive well that will a waste to the state economy and the nation at large.

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